האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Syllabus

Representations of compact Lie groups and Lie algebras - 80971

Last update 01-08-2023

HU Credits: 3

Responsible Department: Mathematics

Academic year: 0

Semester: 2nd Semester

Teaching Languages: Hebrew

Campus: E. Safra

Course/Module Coordinator: Dr. Alexander Yom Din

Coordinator Email: alexander.yomdin@mail.huji.ac.il

Coordinator Office Hours: By appointment

Teaching Staff:
Dr. Yom Din Alexander

Course/Module description:

The defining vector of the course will be to understand a proof of the Weyl character formula using Verma modules, as well as to understand the required background. A very general and tentative description is as follows.

In the first part, we will discuss compact Lie groups, such as SO(3), and their finite-dimensional representations (sample motivation: spectral decomposition of the space of functions on the sphere - the theory of spherical harmonics). We would like to arrive to the problem of classification of irreducible representations and explicit description of their characters (this is Weyl's character formula).

In the second part, we will explain the passage from compact Lie groups and their finite-dimensional representations to semisimple Lie algebras and their finite-dimensional representations (motivation: the latter are "linear" objects, and so easier to understand than the former). We will discuss semisimple Lie algebras and their finite-dimensional representations, so that we will have enough background for the third part.

In the third part, we will want to prove Weyl's character formula. There are different proofs, but the one that we would like to present is given by studying a wider class of representations of semisimple algebras, not just the finite-dimensional ones; The class is called "category O", and important representatives in it are "Verma modules". The idea is that Verma modules are simpler than the irreducible representations we started with (despite being infinite-dimensional), and their character is easily calculated. By understanding how an irreducible representation is "glued" from Verma modules, we will obtain Weyl's character formula.

We will try to edit the course so that required background knowledge is not very wide. For example, I will define what a topological group is, what an (embedded) manifold is, what are tangent spaces, etc. There is no required background in Lie groups or representation theory, but I do expect basic background in basic topology and basic analysis in several variables.

Course/Module aims:

Same as in learning outcomes.

Learning outcomes - On successful completion of this module, students should be able to:

Expanding the student's knowledge in the chosen subject.

Developing independent learning skills.

Acquiring the ability to read advanced mathematical texts.

Preparation for research.

Attendance requirements(%):
100

Teaching arrangement and method of instruction: Lecture

Course/Module Content:
-

Required Reading:
-

Additional Reading Material:
None

Grading Scheme:
Essay / Project / Final Assignment / Home Exam / Referat 100 %

Additional information:
Other topics might be taught!