האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Syllabus

FUNDAMENTAL CONCEPTS IN DIFFERENTIAL GEOMETRY - 80608

Last update 01-09-2021

HU Credits: 6

Responsible Department: Mathematics

Academic year: 0

Semester: 1st Semester

Teaching Languages: Hebrew

Campus: E. Safra

Course/Module Coordinator: Jake Solomon

Coordinator Email: jake@math.huji.ac.il

Coordinator Office Hours: By appointment.

Teaching Staff:
Prof Or Hershkovits,
Mr. Leder Roee

Course/Module description:
The foundations of differential geometry.

Course/Module aims:
The students will get familiar with the basic terms and tools of differential geometry and will be able to formulate and solve problems in this area. Additionally, students will be able to use the language developed to study more advanced topics in the field.

Learning outcomes - On successful completion of this module, students should be able to:

1. Formulate and prove main theorems lying at the foundations of differential geometry.

2. Apply tools of differential geometry to related fields, such as smooth dynamics and Lie group actions, hyperbolic geometry.

3. Explain important terms in differential and Riemannian geometry, and in particular, suggest various interpretations of the Riemannian curvature tensor.

4. Deduce topological conclusions from geometric data.

5. Interpret smooth analytical statements geometrically and vice versa.

Attendance requirements(%):
While there is no formal attendance requirement, students are expected to learn the content of lectures, which may not be available from any of the course reading. Moreover, students are expected to be aware of any announcements made in lecture.

Teaching arrangement and method of instruction: Lecture + exercise

Course/Module Content:
The first part of the course is devoted to presenting the central concepts: differentiable manifolds, transversality, vector fields, Lie groups, differential forms, integration on manifolds, the generalized Stokes' theorem, Riemannian metrics, connections, geodesics and Riemannian curvature.

The continuation of the course treats various interpretations of Riemannian curvature: the relation between Riemannian curvature and divergence of geodesics via the Jacobi equations, the relation between Riemannian curvature and curvatures of curves via the second fundamental form and Gauss' theorem, and the connection between curvature and the energy functional via the calculus of variations.

To conclude, the course discusses the influence of curvature on topology: Hadamard's theorem on the triviality of the higher homotopy groups of a manifold with negative curvature, the Bonnet-Myers theorem on finiteness of the fundamental group in positive Ricci curvature, and Synge-Weinstein theorem on simple connectedness in even dimensions for positive curvature. Further topics may be covered if time permits.

Required Reading:
none

Additional Reading Material:
Do Carmo, "Riemannian Geometry"

Lee, "Introduction to Smooth Manifolds"

Do Carmo, "Differential Geometry of Curves and Surfaces"

Bott, Tu, "Differential forms in Algebraic Topology"

Warner, "Foundations of Differentiable Manifolds and Lie Groups"

Grading Scheme:

Additional information: