# The Hebrew University of Jerusalem

## Syllabus

# Open problems in mathematics - 80530

Last update 27-04-2024

HU Credits: 3

Degree/Cycle: 1st degree (Bachelor)

Responsible Department: Mathematics

Academic year: 0

Semester: 2nd Semester

Teaching Languages: English

Campus: E. Safra

Course/Module Coordinator: Elon Lindenstrauss

Coordinator Email: elon.bl@mail.huji.ac.il

Coordinator Office Hours: By appointment

Teaching Staff:

*Prof Elon Lindenstrauss*

*Course/Module description:*

*The course will present, in a way accessible to third year undergraduate students, 2-3 famous open problems in mathematics, including background and some noteworthy related results.*

*The course will be in English.*

*Among the problems that may be discussed*

* *Riemann Hypothesis*
* *Littlewood Conjecture on Diophantine approximations*
* *Kakeya Conjecture*

*Course/Module aims:*

*The aim of the course is to present 2-3 well-known open problems in a way that is accessible to Advanced undergraduate students. along the way important Concepts in modern mathematics will be introduced in a motivated way.*

*Learning outcomes - On successful completion of this module, students should be able to:*

*After finishing the course, students will*
*learn about three important open problems*
*will have some insight about the kind of problems a mathematician investigate*
*will learn important concepts such as modular forms, lattices*

*Attendance requirements(%):*

*85*

*Teaching arrangement and method of instruction: two hours frontal teaching + hour of open discussion*

*Course/Module Content:*

*The problems I hope to discuss are the following:*

*1. **The Riemann hypothesis**. Probably the most famous problem in mathematics, and by a wide margin (P&eq;NP is arguably as famous, and possibly of more practical importance, but is somewhere between theoretical computer science and mathematics). For such a famous problem, it takes some amount of efforts to state (the usual definition of the Riemann zeta function makes sense only for $Re s > 1$!), and it is even harder to explain why it is important. I will pursue a somewhat nonstandard (or at least not standard for presentation of the Riemann hypothesis from an elementary point of view) emphasizing the connection between the Riemann hypothesis and the quotient of the hyperbolic plan $H &eq; left{ z: Im z > 0
ight}$ by the group $SL (2, □)$ acting on $H$ by Mobius transformations. Some references we will use include [@Titchmarch-book, @Zagier-Eisenstein-and-Riemann, @Zagier-short-proof]*

*1. **The Littlewood conjecture**. This conjecture states that for every $alpha, □eta in R$,
$$
liminf _ {n o infty} n sin (n alpha) sin (n □eta) &eq; 0
.$$
This is a problem I worked on myself [@Einsiedler-Katok-Lindenstrauss], though we will mostly discuss a much older paper [@Cassels-Swinnerton-Dyer] that aged very well!*

*1. **The Kakeya conjecture**. This conjecture states that for any (say) compact subset of $R ^ d$ which contain a unit segment of a line in every direction has to have Hausdorff dimension $geq d$. This conjecture is known for $d&eq;2$; moreover it is easy to see that for any $d$ such a set has to have dimension $geq 1+ frac {d-1}2$. There are stronger bounds for every $d geq 3$ (see e.g. [@Wolff-Kakeya]). One can ask an analogous problem also over a finite field. To the surprise of many, the question of a finite fields has a complete solution by Dvir [@Dvir-Kakeya].*

*Required Reading:*

*Reading assignments may be given during the semester*

*Additional Reading Material:*

*Bibliography*

*1. Titchmarsh, E.C.: The theory of the Riemann zeta-function. The Clarendon Press, Oxford University Press, New York (1986)*

*2. Zagier, D.: Eisenstein series and the Riemann zeta function. In: Auto- morphic*

*forms, representation theory and arithmetic (Bombay, 1979). pp. 275–301. Springer, Berlin-New York (1981)*
*3. Zagier, D.: Newman's short proof of the prime number theorem. Amer. Math. Monthly. 104, 705–708 (1997). https://doi.org/10 .2307/2975232 2*
*4. Einsiedler, M., Katok, A., Lindenstrauss, E.: Invariant measures and the set of exceptions to Littlewood's conjecture. Ann. of Math. (2). 164, 513–560 (2006)*
*5. Cassels, J.W.S., Swinnerton-Dyer, H.P.F.: On the product of three homogeneous linear forms and the indefinite ternary quadratic forms. Philos. Trans. Roy. Soc. London. Ser. A. 248, 73–96 (1955)*
*6. Wolff, T.: Recent work connected with the Kakeya problem. In: Prospects in mathematics (Princeton, NJ, 1996). pp. 129–162. Amer. Math. Soc., Providence, RI (1999)*
*7. Dvir, Z.: On the size of kakeya sets in finite fields. Journal of the American Mathematical Society. 22, 1093–1097 (2009)*

*Grading Scheme:*

*Essay / Project / Final Assignment / Home Exam / Referat 100 %*

*Additional information:*

*Homework assignment will be given during the semester. they will not be graded, but some of these problems will be included in the take-home final*