האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

The Hebrew University of Jerusalem

Syllabus

INFINITESIMAL CALCULUS (2) - 80132

Last update 26-02-2014

HU Credits: 7

Degree/Cycle: 1st degree (Bachelor)

Responsible Department: Mathematics

Academic year: 1

Semester: 2nd Semester

Teaching Languages: Hebrew

Campus: E. Safra

Course/Module Coordinator: Dr. Dan Mangoubi

Coordinator Email: mangoubi@math.huji.ac.il

Coordinator Office Hours: By appointment

Teaching Staff:

Prof Azriel Levy
Dr. Dan Mangoubi
Dr. Yves Godin
Latif Eliaz
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Course/Module description:

1. Integration and the Riemann integral (Darboux's approach). Riemann's criterion for integrability of functions. Families of integrable functions: monotonic functions and continuous functions. Riemann sums. Definition of the integral by Riemann sums. Notion of variation. The fundamental theorem of calculus. Newton-Leibnitz formula. Methods of definite integration: substitution and integration by parts. The indefinite integral: primitives. Integration techniques: by parts and substitution. Integration of rational functions. Integral forms of the remainder in Taylor's theorem. Applications of integrals to geometric and physical problems. Numerical integration. Improper integrals: integrals of unbounded functions, integrals on unbounded intervals. Convergence theorems. Absolute and conditional convergence.

2. Series of real numbers. The sequence of the partial sums. Convergent series. A sufficient condition. Cauchy's criterion for the convergence of series. Tails and remainders. Series with positive terms. The comparison test, the quotient test, the root test and the condensation test. . Series with terms of alternating signs. Leibniz's theorem. Dirichlet's criterion and Abel's theorem. Absolute convergence and conditional convergence. Associativity and commutativity in a series. Riemann's theorem on the rearrangement of a conditionally convergent series. The product of absolutely convergent series. The Cauchy product. Applications: definition of the trigonometric functions, decimal representation of the real numbers.

3. Sequences and series of functions. Pointwise and uniform convergence. Cauchy criterion. Uniform convergence and continuity. Dini's theorem. Uniform convergence and integration. Uniform convergence and differentiation. Power series. Region and radius of convergence. Behavior at boundary and Abel's theorem. Power series expansions of elementary functions. Analytic functions.

4. Plane curves. Support of a plane curve. Limits and continuity. Operations: sum, product with a real-valued function, scalar product, composition with a real-valued function (parametrisation). Differentiable curves and tangent vector. Arithmetic of derivatives. The chain rule. Smooth plane curves. Real- valued functions of bounded variation. Arc length. Parametrisation by arc length .

Course/Module aims:

Same as in learning outcomes.

Learning outcomes - On successful completion of this module, students should be able to:

Ability to prove and apply the theorems presented in the course.

Ability to apply correctly the mathematical methodology in the context of the course.

Acquiring the fundamentals as well as basic familiarity with the field which will assist in the understanding of advanced subjects.

Ability to understanding and explain the subjects taught in the course.

Attendance requirements(%):

0

Teaching arrangement and method of instruction: Lecture + exercise

Course/Module Content:

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Required Reading:
None

Additional Reading Material:
None

Course/Module evaluation:
End of year written/oral examination 0 %
Presentation 0 %
Participation in Tutorials 0 %
Project work 0 %
Assignments 0 %
Reports 0 %
Research project 0 %
Quizzes 0 %
Other 100 %

Additional information:
Composition of the final grade will be announced at the beginning of the course.