The Hebrew University of Jerusalem

Syllabus

Mathematical Methods I - 80114

Last update 03-10-2021

HU Credits: 6

Degree/Cycle: 1st degree (Bachelor)

Responsible Department: Mathematics

Academic year: 0

Semester: 1st Semester

Teaching Languages: Hebrew

Campus: E. Safra

Course/Module Coordinator: Prof Ruth Lawrence-Naimark

Coordinator Email: ruthel.naimark@mail.huji.ac.il

Coordinator Office Hours: by prior arrangement

Teaching Staff:

Dr. Miriam Bank,
Dr. Moriah Sigron,
Mr. Leder Roee,
Mr. Cohen Omri

Course/Module description:

The aim of the course, along with its second semester companion, is to supply all the basic mathematical tools (apart from linear algebra) for science students (primarily physics, engineering and computer engineering) for first degree studies. This course covers vectors, complex numbers, coordinate systems, differentiation and integration of functions of one variable, Taylor's theorem, differentiability of functions of several variables, double and triple integrals, vector fields, integration over curves and surfaces of scalar and vector fields, with an emphasis on intuitive understanding and use rather than rigor.

Course/Module aims:

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Learning outcomes - On successful completion of this module, students should be able to:

Being able to work freely with the basic concepts of Calculus and geometry.

Acquiring the ability to analyze a more complicated problem which involves the use of several tools simultaneously.

Familiarity with basic notions in mathematics.

Familiarity with some of the mathematical tools used in the exact sciences.

Attendance requirements(%):
None

Teaching arrangement and method of instruction: The core material is methodically presented in frontal lectures, with minimal examples only in order to illustrate the ideas as presented. The student is expected to read the lecture summaries both before and after classes and then do the recommended homework examples, on their own or with the help of solutions provided. It is expected that the student spend around 2 hours out of class for each hour of frontal presentation, working with the material and doing problems.

Course/Module Content:
1: Vectors and geometry of R^n

Vector as length and direction; position vector OP, vector AB
Coordinates in R^n
Operations +,c., || ||, dot product, angle, "work done"
Parametric form of line and plane (Cartesian coordinates and vectorially)
r.n&eq;p (meaning of n,p)
vector product in R^3: in coordinates and geometrically (a.b.sin theta); "torque"
triple scalar product as volume
2x2, 3x3 determinants; calculation and meaning as areas/volumes

2: Coordinate systems and some elementary geometric objects

(cos t, sin t) as parametric form for unit circle in R^2;x^2+y^2&eq;1
parametric form and Cartesian equation for ellipse in standard form
y&eq;x^2, xy&eq;c, x^2-y^2&eq;c
polar coordinates in plane; examples
cylindrical and spherical polar coordinates in R^3; examples

3: Complex numbers
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formal symbol i; +,x,/ of numbers a+ib
Argand plane, modulus, argument and conjugate
cis; Euler's formula; exponential and log of complex
numbers

4: Functions of one variable (mainly review)
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functions, domain of definition, graph, image

composition of functions and their graphs
graphs of f(x-a),f(x/a) in relation to graph of f(x)
graphical representation of roots of f(x)&eq;a, f(x)&eq;g(x)
elementary functions log, exponential, power, trignometric, hyperbolic
and discussion of their graphs and inverses
intuitive idea of continuity(existence of max/min on closed
bounded interval; intermediate value theorem)

5: Derivatives and integrals (mainly review)

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velocity/distance graphs and their relations
derivative as limit delta s/delta t; definite integral as
limit of sum - Riemann sums (no theory)
graphical meaning as slope/area
example of non-differentiable function
differentiability as linear approx, tangent line
notations for derivative
chain rule; use with implicit function
derivative of sum, product, inverse
tables of derivatives of elementary functions; examples
differentials

6: Taylor's series

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higher derivatives; notations
Taylor's formula for polynomials
Taylor's formula with Lagrange form of remainder (no proof)
geometric meaning for small n
leading and sub-leading approximations
examples for elementary functions sin, cos, ln(1+x), 1/(1-x)
arc tan, exp, (1+x)^a
note convergence issue, state region (radius(of convergence
for standard functions only
sketching graphs of functions

7: Integration of functions of one variable

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fundamental theorem of calculus
indefinite integration: integration by parts, substitution,
examples, use of tables; partial fractions; rational
trignometric functions

8: Definite integration

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relation to indefinite integral;
change of variables; examples; idea of numerical methods
improper integrals (simple cases only)
applications to finding areas, volume of body of revolution

9: Functions R->R^n
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function R->R^n as parametric form of curve; graph as
world-line; derivative as velocity vector;
tangent line as linear approximation; differentials
length of curve

10: Functions of several variables R^n->R
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functions, graphs, level curves, sections
directional derivatives, partial derivatives
differentiability as existence of linear approximation;
tangent plane; example of non-differentiable function
gradient, chain rule, geometric interpretation of gradient;
differentials
implicit functions
stationary points, form of level curves near stationary point
form of Taylor's theorem without remainder for function of 2
variables; type of stationary point
Lagrange multipliers, finding global max/min of functions on
compact regions in R^2, R^3

11: Functions R^m->R^n (m,n&eq;2,3)
----
interpretation as vector field (m&eq;n); parametrisation of
surface (m&eq;2,n&eq;3); change of coordinates (m&eq;n);
derivative as linear approximation (tangent map)/matrix
derivative; Jacobian
line integrals
div, curl

12: Integration of functions of several variables
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double and repeated integrals
Fubini's theorem
integration on non-rectangular regions
triple integrals and applications (volumes, centre of mass, moment of
inertia)
change of coordinates (particularly for polar/cylindrical/spherical
coordinates)

13: Integration on surfaces
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definition of surface integral of a vector field (flux) and of a scalar field

Required Reading:
Book of lecture notes and book of exercises and solutions, both available electronically on the MOODLE page of the course and directly from the coordinator in printed form.

Additional Reading Material:
Any book on mathematical methods for science students and/or calculus for science students.

Course/Module evaluation:
End of year written/oral examination 60 %
Presentation 0 %
Participation in Tutorials 0 %
Project work 0 %
Assignments 0 %
Reports 0 %
Research project 0 %
Quizzes 20 %
Other 20 %
midterm

Additional information:
There is a midterm exam (no moed b) in 8th week on the material of the course up to and including 7th week (essentially all the material on vectors, complex numbers and functions of one variable including continuity, differentiation and Taylor series). This exam is NOT compulsory, but gives 20% magen against the final exam, so that for those choosing not to take it, the final exam is 80% of the final grade.

There are also two repeatable computerized "gateway" tests, one on differentiation and the other on simple integration techniques, each worth 5% of the final course grade if passed.

The midterm and final exams will be held on campus if possible and in a similar form from a distance, if not.

The weekly quizzes in the course are taken via computer. Every week when there is no other major test, there is a computer graded online multiple-choice quiz and

additionally a randomly-generated assignment, the solutions to which students submit on-line, consisting of 2 or 3 problems of the type which might arise in the final exam, connected with the material of the previous week in the course lectures. Combined, these contribute 10% to the final course grade.
Other or additional topics may be studied